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Investigation of the Performance of Synchronous Generators Equipped with Nonlinear Excitation Controller

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Abstract— Investigation of the dynamic performance of a synchronous generator connected to an infinite bus (SMIB) system is carried out in this paper. The generator is equipped with a n onlinear excitation control law based on the concepts of geometric homogeneity and feedback linearization. A new positive parameter, called the dilation gain, is introduced in the control law for improved damping of oscillations and better dynamic performance. Two models of the system are employed for the study, and a disturbance in form of a network fault with varied durations is applied to test the performance of the system. Simulation results as well as MATLAB® code for testing for exact linearization of an affine nonlinear system are provided.

Index Terms— exact linearization, fault cycles, finite-time stability, homogeneity, nonlinear control laws, power system model

1 INTRODUCTION

A typical modern power system generally consists of a large number of generating electric power sources (mostly synchronous generators) interconnected through complex networks of transmission lines with myriad automatic and protection equipment pieces for the sole purpose of meeting the power demands of a large number of different loads. It has thus been considered as "... a high-order multivariable system whose dynamic response is influenced by a wide array of devices with different characteristics and response rates" [1]. In this complex and highly dimensioned system, in order to accommodate more load demands and provide a constant and reliable electric power supply, power system controllers are employed (and constantly being encouraged) at the generation, transmission and distribution levels to deliver electric power to the load centers efficiently. Besides, generator excitation system controllers have been recognized as one of the most reliable and economic way of damping power system oscillations and improving the overall system stability [2]. Local plant and inter-area mode oscillations occur in power systems, and pose major challenges to power system control engineers. These oscillations are usually caused by lack of sufficient generator rotor damping torque (and this phenomenon characterized the earliest exciter/AVR due to the increase in bandwidth associated with the AVR loop) [3], [4].

The challenges become more stringent as power systems undergo changes due to network alterations (caused by faults or switching events) and/or variations in loads. Owing to the availability of powerful and low-cost computing resources, which has spurred the design, development, investigation, and complete analysis of nonlinear control algorithms, as is evident in many practical implementations [5], [6], [7], there has been dedicated research in feedback linearization control (FBLC)—a significant area of application of nonlinear control techniques for power system stabilization. FBLC involves complete or partial transformation of nonlinear systems into equivalent linear ones that are amenable to linear control design techniques [8], [9]. Several versions of FBLC have been applied to the design of power system excitation control [10], [11], [12], [13]. For instance, Gan *et al*. [11] proposed an improved FBLC using a linear optimal state-space feedback and saturation-type nonlinear robust control strategies—here rotor angle oscillations were damped out in about 15s after perturbing system under the action of the proposed controller. More recently, Mahmud *et al*. [14] proposed a zero dynamics-based excitation controller which was able to remove (in about 1.8-2s) rotor angle oscillations due to a three-phase fault that lasted for about 0.2s. In this paper, a combination of the concepts of geometric homogeneity and feedback linearization is employed to construct a nonlinear excitation controller which has the ability to damp out rotor angle oscillations within 2s for a three-phase fault with as much as 0.3-s duration.

In the rest of the paper, Section 1 presents the model of the power system for the study, while the construction of the controller is described in Section 3. In Section 4, results of system simulations are provided and discussed, and the concluding comments are given in Section 5.

2 POWER SYSTEM MODEL

Two models of the power system, based on the single machine connected to an infinite bus (SMIB) system shown in Fig. 1, are employed for the work presented in this paper. They are the one-axis (third-order) and two-axis (fourth-order) models which give approximate descriptions of the dynamic performance equations of the system. The latter is particularly considered to further show the performance of the controller under identical network conditions imposed on the former.

The third-order model is given by [15], [16]

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{1}$$

$$\frac{d\omega}{dt} = A_1 + \frac{F_2 V^2}{2}\sin 2\delta - A_4 V E'_q \sin\delta - \underbrace{T'_{q0}\frac{F_3 F_1}{M} V^2 \cos^2\delta}(\omega - \omega_S) \tag{2}$$

$$\frac{dE'_q}{dt} = -B_1 E'_q + B_2 V\cos\delta + \frac{1}{T'_{do}} E_f \tag{3}$$

while the fourth-order model is given by [15], [17], [18]

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{4}$$

$$\frac{d\omega}{dt} = \frac{T_m}{M} - \frac{1}{2}A_1 V^2 \sin 2\delta + A_2 V E'_d \cos\delta - A_3 V E'_q \sin\delta \tag{5}$$

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$$\frac{dE_q'}{dt} = -B_1 E_q' + B_2 V\cos\delta + \frac{1}{T_{do}'} E_f \quad (6)$$

$$\frac{dE_d'}{dt} = -\frac{E_1}{T_{qo}'} E_d' + \frac{E_2}{T_{qo}'} V\sin\delta, \quad (7)$$

where δ is the rotor or torque angle in radians, ω is the rotor speed in radians/s, E_q' is the q-axis voltage which is proportional to the field winding flux linkage, E_d' is the d-axis voltage which is proportional to the amortisseur winding flux linkage, Ef represents the excitation coil voltage, V is the magnitude of the voltage of the infinite bus, ωs is the synchronous speed of the generator, Tm is the input torque, and M (= 2H/ωs), is the moment of inertia, where H is the generator inertia constant in seconds.

The parameters A1-A3, B1, B2, E1, and E2 are defined as follows:

$$A_1 = \left(\frac{1}{X_q'+X_E} - \frac{1}{X_d'+X_E}\right)\frac{1}{M};\ A_2 = \left(\frac{1}{X_q'+X_E}\right)\frac{1}{M};\ A_3 = \left(\frac{1}{X_d'+X_E}\right)\frac{1}{M}$$

$$B_1 = \frac{(X_d+X_E)}{T_{do}'(X_d'+X_E)};\ B_2 = \frac{(X_d-X_d')}{T_{do}'(X_d'+X_E)}$$

$$E_1 = \frac{(X_q+X_E)}{(X_q'+X_E)};\ E_2 = \frac{(X_q-X_q')}{(X_q'+X_E)}$$

$$F_1 = \frac{(X_q-X_q')}{(X_q+X_E)};\ F_2 = \left(\frac{1}{X_d'+X_E} - \frac{1}{X_q+X_E}\right)\frac{1}{M};\ F_3 = \left(\frac{1}{X_q+X_E}\right).$$

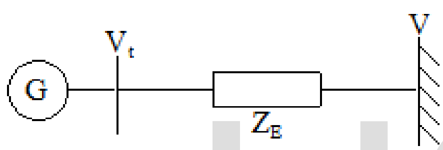

Fig. 1. Representation of a SMIB

3 CONTROL LAW CONSTRUCTION

Consider an affine nonlinear power system represented by the model

$$\dot{x} = f(x,t) + g(x)u, \quad (8)$$

where x is the system state vector, f and g are continuously differentiable functions, and u is the control signal. The control law construction consists in 1) obtaining an output signal such that the system can be exactly or partly feedback-linearized, and that the system internal dynamics, if any, remain asymptotically stable; and 2) deriving a nonlinear control law, u, that will ensure that the output signal becomes zero in finite time and remains so thereafter under both normal and disturbance-induced conditions.

3.1 System Linearization

Using a suitable system output function y(x), the system given in equation (8) can be exactly linearized into the Bruvnosky normal form as

$$dz_1/dt = z_2 \quad (9)$$

$$dz_2/dt = z_3 \quad (10)$$

$$\vdots$$

$$dz_n/dt = h, \quad (11)$$

with h given by

$$h = f(z) = L_f^n y + L_g L_f^{n-1} yu, \quad (12)$$

where n is the order of the system, and $L_g L_f^i y(x)$ represents the Lie derivative of $L_f^i y(x)$ along the function g(x).

The exact linearization condition can be determined for any affine SISO nonlinear system using the flowchart shown in Fig. 2 as well as the MATLAB code given in Appendix A (which can be readily extended to a MIMO system). The chart is based on Definition 1 given below.

Definition 1 [19]: Consider the nonlinear system in equation (8). The system can be exactly linearized if its order n equals its relative degree r. This condition is satisfied if the matrix

$$P = [g(x) \quad ad_f g(x) \quad ad_f^2 g(x) \quad \cdots \quad ad_f^{n-1} g(x)] \quad (13)$$

has rank n near the system operating point, x0, and the matrix

$$D = [g(x) \quad ad_f g(x) \quad ad_f^2 g(x) \quad \cdots \quad ad_f^{n-2} g(x)] \quad (14)$$

involutes at x = x0. The involutivity condition is that matrix D and any of its variant

$$D_s = \left[g(x) \quad \cdots \quad ad_f^{n-2} g(x) \quad \left[ad_f^i g(x), ad_f^j g(x)\right]\right] \quad (15)$$

have rank n-2, where i = 1, 2, …, n-2, j = 1, 2, …, n-2, and i ≠ j. The symbol $ad_f g(x)$ or [f(x), g(x)] is called the Lie bracket of g(x) along f(x), and $ad_f^i g(x) = ad_f\left(ad_f^{i-1} g(x)\right)$.

Various output functions (measurable and/or convenient) can be chosen and then tested using the MATLAB code.

3.2 System Controller Derivation

The overall control law is now obtained based on the concept of geometric homogeneity. Simply stated, homogeneity is the feature of functions and vector fields associated with dynamic systems, which guarantees their transformation (dilation) from one point to another in the state space.

Generally, system dilation is in the form

$$\Delta_e(z) = (e^{m_1} z_1, e^{m_2} z_2, \cdots, e^{m_n} z_n), \quad (16)$$

which is an extension of the standard dilation [20]

$$\Delta_e(z) = (ez_1, ez_2, \cdots, ez_n). \quad (17)$$

Therefore, if the system given in equations (9)-(11) is dilated, then h in equation (12) becomes

$$h = f(e^m z) \text{ or } h = f(ez). \quad (18)$$

This concept is employed to modify the finite-time stabilizing feedback controller presented in [20] (Proposition 8.1), and given as follows: Consider the system defined in equations (9)-(11). There exists a feedback control law

$$h(ez) = -k_1 \mathrm{sign}(ez_1)|ez_1|^{v_1} - \cdots - k_r \mathrm{sign}(ez_r)|ez_r|^{v_r} \quad (19)$$

which ensures that the origin is globally finite-stable, where e > 0 is called the dilation gain. The positive numbers $k_1, k_2, \ldots, k_r$ are appropriately selected such that the polynomial

$$p^r + k_r p^{r-1} + k_{r-1} p^{r-2} + k_1$$

is Hurwitz.

v1, v2, …,vr are found from

$$v_{i-1} = \frac{v_i v_{i+1}}{2v_{i+1} - v_i}, \quad i = 2, 3, \cdots, r$$

with

$$v_{r+1} = 1;\ v_r \in (1-\varepsilon, 1);\ \varepsilon \in (0,1).$$

Thus, by combining equations (12) and (19), the control law yields

$$u = \frac{h(ez)|_{z=\Phi(x)} - L_f^n y}{L_g L_f^{n-1} y}, \quad (20)$$

where Φ(x) is a diffeomorphism which maps the system from x-domain into z-domain and vice versa.

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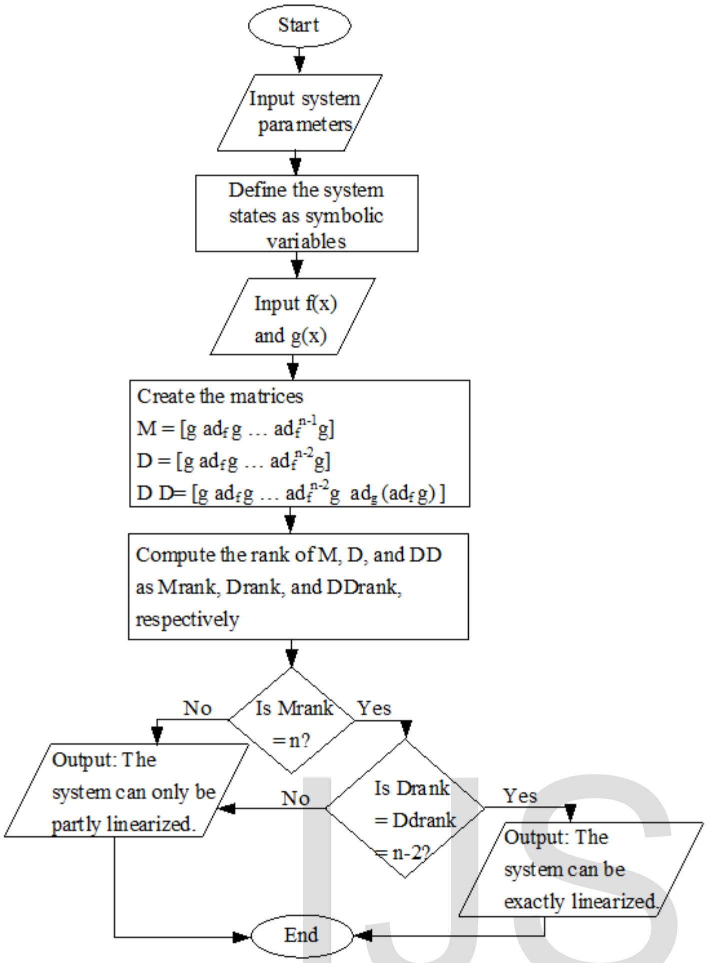
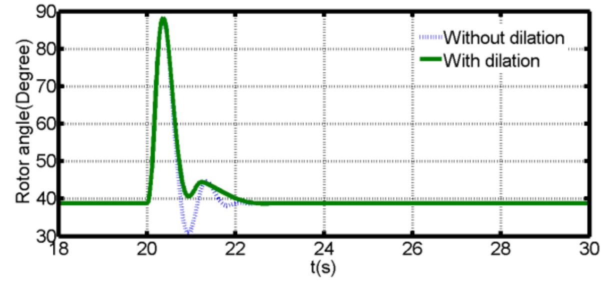


Fig. 2. Flowchart for testing the exact linearization condition for a general affine SISO nonlinear system

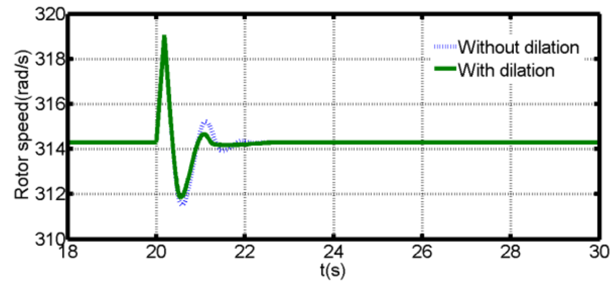
4 SIMULATION RESULTS AND DISCUSSION

The graphical results showing the system variables for various cycles of a three-phase symmetrical fault applied at the generator terminals are displayed in Figs. 3-6. The waveforms indicate a performance comparison of the system in terms of dilation (with the gain of 5) and non-dilation (with the gain of unity) of the control signal. The values of all the system parameters, including the controller parameters, are provided in Appendix B.

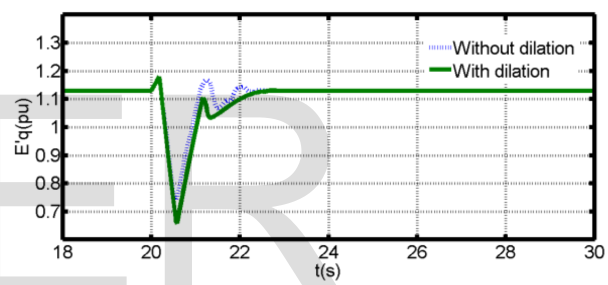
In all the figures, it is clear that the effect of a positive multiplicative gain is to minimize oscillations, thereby improving the system damping—in particular, Fig. 3(a) and Fig. 4(a) show this effect more clearly, as the control activity (displayed in Fig. 3(d) and Fig. 4(d)) reflects. But, as Fig. 5 depicts, the dilation gain weakens the controller ability to make the system withstand longer fault duration, such as 15cycles. As long as the fault cycle does not exceed 14.5cycles (0.29s), which is relatively high, the effect of the gain remains constructive (see Fig 4). To further highlight the benefit of the gain, Fig. 6 shows a similar set of waveforms for a fourth-order SMIB system subjected to the same fault for duration of 0.18s.



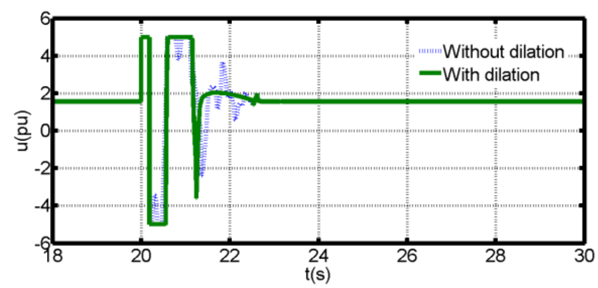
(a) Rotor angle



(b) Rotor speed

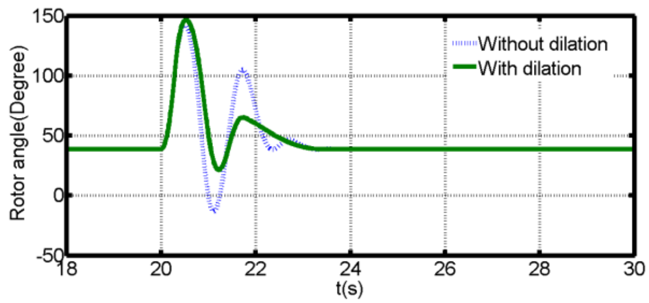


(c) Quadrature EMF

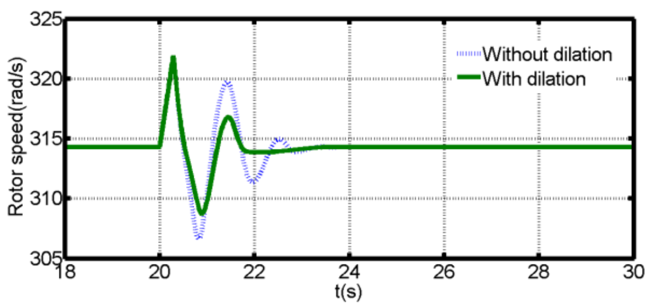


(d) Control effort

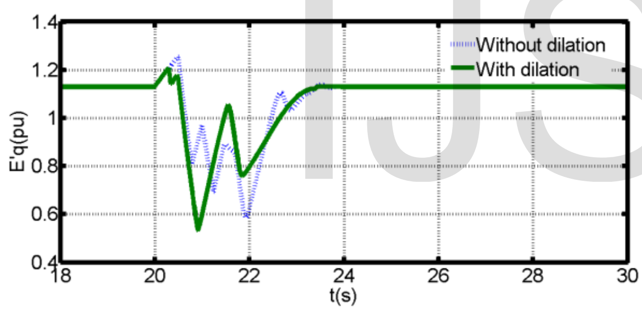
Fig. 3: Waveforms showing the effect of the dilation gain for a generator terminal fault cleared after 9cycles (third-order SMIB)



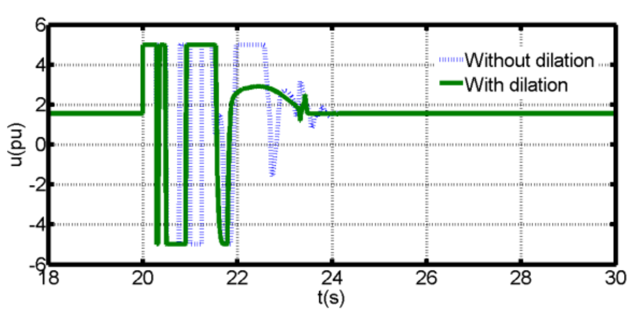
(a) Rotor angle



(b) Rotor speed

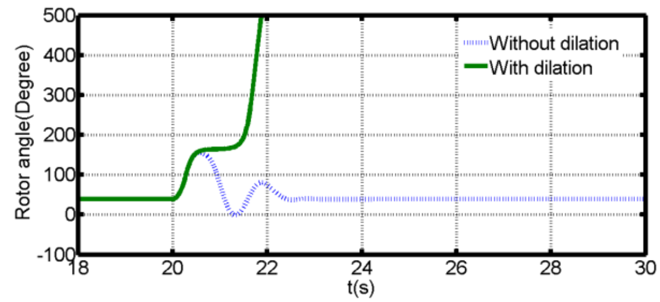


(c) Quadrature EMF

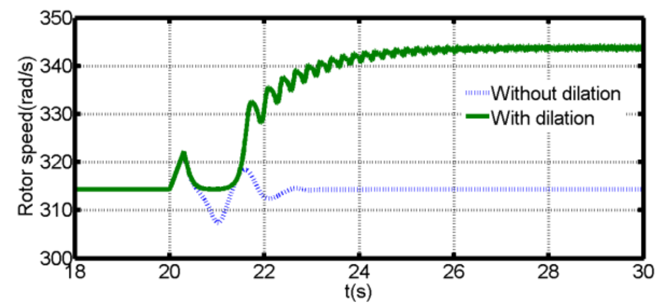


(d) Control effort

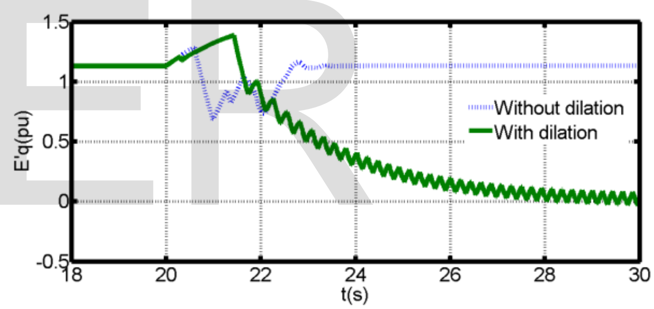
Fig. 4: Waveforms showing the effect of the dilation gain for a generator terminal fault cleared after 14.5cycles (third-order SMIB)



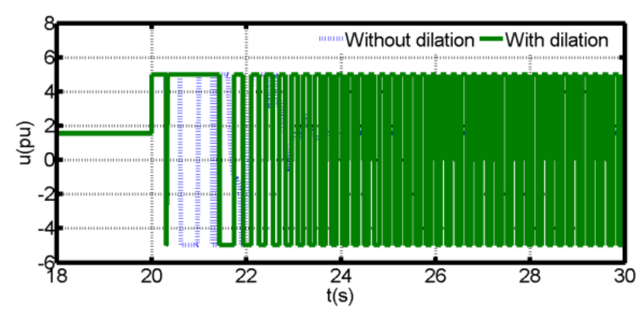
(a) Rotor angle



(b) Rotor speed



(c) Quadrature EMF



(d) Control effort

Fig. 5: Waveforms showing the effect of the dilation gain for a generator terminal fault cleared after 15cycles (third-order SMIB)

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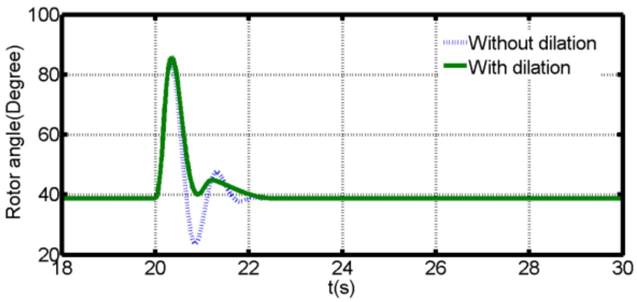
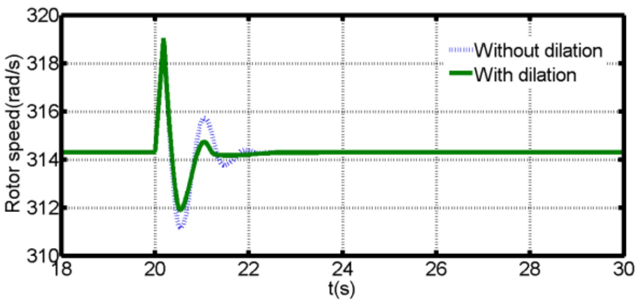
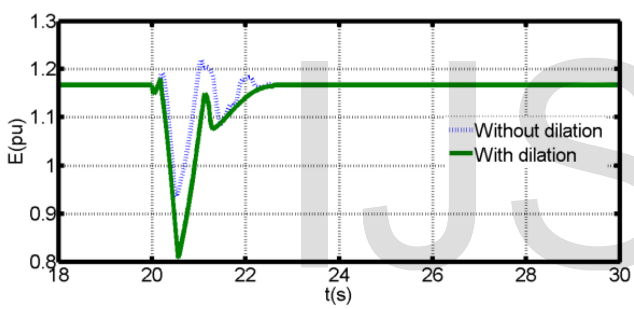


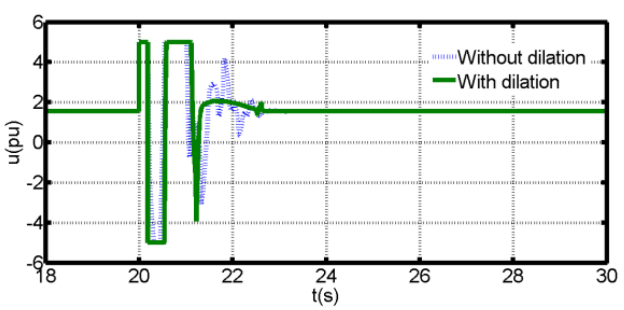
(a) Rotor angle



(b) Rotor speed



(c) Resultant EMF



(d) Control effort

Fig. 6: Waveforms showing the effect of the dilation gain for a generator terminal fault cleared after 9cycles (fourth-order SMIB)

5 CONCLUSION

Investigation of the performance of synchronous generators equipped with nonlinear excitation controller constructed based on the concepts of geometric homogeneity and feedback linearization has been the thrust of this paper. Waveforms demonstrating the controller performance when the system is disturbed are shown. It is established that dilating the control signal has the benefit of enhancing system damping. It is important to mention that realizing this type of excitation control system will require the use of fast power electronic devices.

Appendix A

MATLAB Code for Testing the Exact Linearization Condition for a General Affine Nonlinear System

%This function OutputResult=ELCOND(F,G,S)is used to determine the exact linearization %conditions for any given affine nonlinear SISO system dX/dt=f(X) + g(X)u, where X %represents the states (x1, x2,...,xn) of the system. F, G, and S are symbolic expressions for %f(x),g(x), and the states, respectively. OutputResult is a vector of string elements stating %whether the system can be exactly linearized or not. Note that the order of the system must be %at least 2. ALSO, NOTE THAT THE STATES IN F AND G APPEAR AS x1, x2, x3,..., xn, %WITH THESE , OF COURSE, HAVING BEEN DEFINED AS SYMBOLIC VARIABLES. %For example, the system dx(1)/dt=x(1)sin x(2)+20x(1)-2u and dx(2)/dt=cos x(1)+ 10u having %steady-state values x0(1)=0.5 and x0(2)=2 is created as: syms x1 x2 f g
% f=[x1*sin(x2)+ 20*x1 cos(x1)+10]';g=[-2 10]';x=[x1 x2]';

```
Function  OutputResult=elcond(f,g,x)
sysorder=length(f); d=sysorder-1;
m=zeros(sysorder,sysorder); dd=zeros(sysorder,d);
M= sym(m); D=sym(dd);
f_diff=jacobian(f,x);
M(:,1)=g;
% Compute the elements of M
for k=2:sysorder
   M(:,k)=(jacobian(M(:,k-1),x)*f)-(f_diff*M(:,k-1));
end
% Compute the elements of D and De
if d==1;
D(:,d)=g;
else
for  i=2:d;
D(:,i)=M(:,i);
end
D(:,1)=g;
   De=D;
   De(:,sysorder)=jacobian(D(:,2),x)*D(:,1)-jacobian(D(:,1),x)*D(:,2);
end
% Check for the exact linearization conditions
input('Enter all the n steady-state values as : x1 =  ; x2 = ; x3 =  ; ... ; xn =  ;   ')
input('Enter all the system parameters if any or press the return key      ')
M_comp=subs(M);D_comp=subs(D);De_comp=subs(De);
M_rank=rank(M_comp);D_rank=rank(D_comp);De_rank=rank(De_comp);
if d==1;
if M_rank==sysorder;
OutputResult='The system can be exactly linearized, i.e., there is an output function that makes the system relative equal to the system order ';
else
OutputResult='The system cannot be exactly linearized, i.e.,an
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output function does not exist to make the system relative equal to the system order ';
end
else
if M_rank==sysorder && D_rank==De_rank;
OutputResult='The system can be exactly linearized, i.e., there is an output function that makes the system relative degree equal to the system order';
else
OutputResult='The system cannot be exactly linearized, i.e., an output function does not exist to make the system relative degree equal to the system order';
end
end
```

Appendix B

Typical values for the system parameters employed for this study are given in Table 1 below [16], [17].

Table 1. SMIB Typical Parameters

Synchronous reactance:	Xd = 0.9 p.u.; Xq = 0.7 p.u.
Transient reactance:	X'd= 0.2 p.u.; X'q= 0.2 p.u.
Open-circuit transient time constant:	T'do= 5.00 s; T'qo = 0.13 s
Inertial constant:	H = 5.00 s
Input torque:	Tm = 0.8413
Transmission line reactance:	XE = 0.24 p.u.
Transformer reactance:	XT = 0.13 p.u.
Infinite-bus voltage magnitude:	V = 1.0 p.u.

Also, the parameters k_1, k_2, and k_3 of h(ez) given in equation (19) are found using the pole-placement method from

$$p^3 + k_3 p^2 + k_2 p + k_1 = (p + a_1)(p + a_2)(p + a_3) = 0 \qquad \text{(A1)}$$

where $a_1 = 9$, $a_2 = 5$, and $a_3 = 2$. Thus, $k_1 = 90$, $k_2 = 73$, and $k_3 = 16$; the value of parameter v3, from which $v_1 = 1/2$ and $v_2 = 3/5$ are obtained, is 3/4. The value of the dilation constant e is 5.

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